:

New Features of the Mandelstam-Leibbrandt Lightcone Gauge

A. Andraši
'Rudjer Bošković' Institute, Zagreb, Croatia

Abstract

This is about new unexpected features of the Mandelstam-Leibbrandt prescription found as applied to spacelike Wilson lines. The regularization parameter ω in the M-L denominator for the spurious poles has to be kept throughout the calculation till the very end or else the integrals do not make sense. We get various 'ambiguous' terms of the form $\omega^{-\frac{\epsilon}{2}}\epsilon^{-2}$ which are not controlled by any sort of Ward identity. These terms cancel out in the sum and the final result is independent of ω. However, for the selfenergy on the spacelike Wilson line we obtain an unexpected double pole at $\epsilon = 0$, using dimensional regularization in $4 - \epsilon$ dimensions.

Electronic address:(internet)andrasi@thphys.irb.hr

1. Introduction

Non-covariant gauges found applications in a wide range of fields, such as QCD, supersymmetric Yang-Mills and string theories. A favourite among them is Mandelstam's lightcone gauge [1] defined by two lightlike vectors satisfying

$$n^2 = n^{*2} = 0, \quad n \cdot n^* = 2, \tag{1}$$

and the propagator

$$\frac{\delta_{ab}}{k^2 + i\eta}\{g_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{n \cdot k + i\omega n^* \cdot k}\}. \tag{2}$$

However, this prescription leads to non-local counterterms [2], does not satisfy the optical theorem [3] and in order to satisfy the Piguet-Sibold equation [4] requires a more precise definition [5]. A convenient laboratory to study the prescription is the Wilson loop [6]. Here we take the triangle Wilson loop with two sides in the direction of the two lightlike vectors and the base in spacelike direction, defined by

$$\begin{aligned} x^1_\mu &= n^*_\mu L - n_\mu L t \\ x^2_\mu &= v_\mu 2L(1-s) \\ x^3_\mu &= n^*_\mu L t \end{aligned} \tag{3}$$

where

$$\begin{aligned} n^*_\mu &= (1,0,0,1) \\ n_\mu &= (1,0,0,-1) \\ v_\mu &= (0,0,0,1) \\ 0 \le\ & t \le 1 \\ 0 \le\ & s \le 1. \end{aligned} \tag{4}$$

2. Self Energy Diagram

There are only two diagrams contributing to order g^2 in the lightcone gauge, the vertex graph where the gluon propagates between the base in the direction v_μ and the side in the direction n^*_μ, and the self energy graph on the base in the direction v_μ. The selfenergy diagram in the momentum space contributes

$$W_2 = ig^2 C_R \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2 + i\eta} \frac{k_-}{k_+ + i\omega k_-} \frac{1}{k_3^2} (\cos 2Lk_3 - 1) \tag{5}$$

where

$$k_- = n^* \cdot k = k_0 - k_3,$$
$$k_+ = n \cdot k = k_0 + k_3. \tag{6}$$

Closeing the contour in the upper k_0 half plane, we meet two poles

$$k_0 = -k + i\eta$$

$$k_0 = -k_3 + 2i\omega k_3 \theta(k_3), \tag{7}$$

which give

$$W_2 = \pi g^2 C_R (2\pi)^{-n} \int dk_3 d^{2-\epsilon} K \frac{1}{k} \frac{k + k_3}{k - k_3 + i\omega(k + k_3)} \frac{1}{k_3^2} (\cos 2Lk_3 - 1)$$

$$-4\pi g^2 C_R (2\pi)^{-n} \int_0^\infty dk_3 \int d^{2-\epsilon} K \frac{1}{K^2 + 4i\omega k_3^2 - i\eta} \frac{1}{k_3} (\cos 2Lk_3 - 1). \tag{8}$$

Naively, one would let $\omega \to 0$ before $\epsilon = 4 - n$, in the integrand. Then, the latter integral in eq.(8) would vanish as a tadpole in the perpendicular momentum K. However, after the introduction of polar coordinates

$$k_3 = k \cos\theta = kx,$$

$$d^{3-\epsilon} k = k^{2-\epsilon} dk (1 - x^2)^{-\frac{\epsilon}{2}} dx \tag{9}$$

and integration over k, the first integral leads to an integral which is not defined for any ϵ (we also change the variable of integration $x^2 = y$). Therefore we have to keep ω in the integrand and choose to evaluate the diagram in the strip

$$1 < \epsilon < 2 \tag{10}$$

in which the same diagram is regularized by ϵ in the Feynman gauge. Then the self energy graph becomes

$$W_2 = g^2 C_R (2\pi)^{1-n} \Gamma(-\epsilon) \cos\frac{\epsilon\pi}{2} (2L)^\epsilon \frac{2\pi^{1-\frac{\epsilon}{2}}}{\Gamma(1 - \frac{\epsilon}{2})} \{A + \frac{1}{2} \int_0^1 dy (1 - y)^{-\frac{\epsilon}{2}} y^{\frac{\epsilon - 3}{2}}\}$$

$$-2g^2C_R(2\pi)^{1-n}E. \tag{11}$$

Here two integrals each contain the factor $\omega^{-\frac{\epsilon}{2}}\epsilon^{-2}$ which has no limit as $\omega \to 0$.

$$A = \int_0^1 dy(1-y)^{-\frac{\epsilon}{2}}y^{\frac{\epsilon-1}{2}}[1-y(1-4i\omega)]^{-1}$$

$$= B(\frac{\epsilon+1}{2}, 1-\frac{\epsilon}{2})\{\frac{\Gamma(\frac{3}{2})\Gamma(-\frac{\epsilon}{2})}{\Gamma(\frac{1}{2})\Gamma(1-\frac{\epsilon}{2})} + (4\omega e^{\frac{i\pi}{2}})^{-\frac{\epsilon}{2}}\frac{\Gamma(\frac{3}{2})\Gamma(\frac{\epsilon}{2})}{\Gamma(\frac{1+\epsilon}{2})}\}, \tag{12}$$

$$E = \int_0^\infty dk_3 \int d^{2-\epsilon}K \frac{1}{K^2+4i\omega k_3^2 - i\eta}\frac{1}{k_3}(\cos 2k_3L - 1)$$

$$= \pi^{1-\frac{\epsilon}{2}}\Gamma(\frac{\epsilon}{2})\Gamma(-\epsilon)\omega^{-\frac{\epsilon}{2}}e^{-\frac{i\pi\epsilon}{4}}\cos\frac{\epsilon\pi}{4}L^\epsilon. \tag{13}$$

However these poles cancel in the sum leaving a double pole in ϵ for the self energy graph on the spacelike line.

$$W_2 = (\frac{g}{2\pi})^2 C_R \pi^{\frac{\epsilon}{2}}\Gamma(-\epsilon)\cos\frac{\epsilon\pi}{2}(2L)^\epsilon \frac{\Gamma(1+\epsilon)}{\Gamma(1+\frac{\epsilon}{2})}\{-\frac{2}{\epsilon}+\frac{1}{\epsilon-1}\} \tag{14}$$

The same graph in the Feynman gauge contains only single poles in ϵ.

3. The Vertex Graph

In order to make the calculations more transparent, we devide the integrand for the vertex graph into three parts,

$$W_1 = W_1^a + W_1^b + W_1^c \tag{15}$$

where

$$W_1^a = -2ig^2C_R\int\frac{d^nk}{(2\pi)^n}\frac{1}{k^2+i\eta}\frac{1}{k_3}\frac{1}{k_+ + i\omega k_-}(e^{ik_+L}-1) \tag{16}$$

$$W_1^b = 2ig^2C_R\int\frac{d^nk}{(2\pi)^n}\frac{1}{k^2+i\eta}\frac{1}{k_3}\frac{1}{k_+ + i\omega k_-}(e^{ik_-L}-1) \tag{17}$$

$$W_1^c = 2ig^2C_R\int\frac{d^nk}{(2\pi)^n}\frac{1}{k^2+i\eta}\frac{1}{k_+ + i\omega k_-}\frac{1}{k_3}(e^{2iLk_3}-1). \tag{18}$$

Again we close the contour in the upper k_0 half plane. The first part of the integrand does not contain any 'ambiguous' poles in the strip eq.(10), and contributes

$$W_1^a = g^2C_R(2\pi)^{1-n}\Gamma(-\epsilon)L^\epsilon e^{\frac{i\pi\epsilon}{2}}\frac{2\pi^{1-\frac{\epsilon}{2}}}{\Gamma(1-\frac{\epsilon}{2})}$$

$$\times\{\frac{2}{\epsilon}2^{-\frac{\epsilon}{2}}[1+\frac{\epsilon^2}{4}(\frac{\pi^2}{12}-\frac{1}{2}\ln^2 2)] + \ln 2 - \frac{5}{24}\epsilon\pi^2\}. \tag{19}$$

The second part of the integrand shows the same features as the self energy graph. After k_0 integration it becomes

$$W_1^b = -g^2 C_R (2\pi)^{1-n} \int_0^\infty dk k^{-1-\epsilon} \int_{-1}^1 dx (1-x^2)^{-\frac{\epsilon}{2}}$$

$$\times \frac{1}{x} \frac{1}{1-x+i\omega(1+x)} (e^{-ik(1+x)L} - 1) \frac{2\pi^{1-\frac{\epsilon}{2}}}{\Gamma(1-\frac{\epsilon}{2})}$$

$$+4g^2 C_R \pi^{2-\frac{\epsilon}{2}} \Gamma(\frac{\epsilon}{2}) \Gamma(-\epsilon) \omega^{-\frac{\epsilon}{2}} e^{\frac{i\pi\epsilon}{4}} L^\epsilon (2\pi)^{-n}. \tag{20}$$

Each integral in eq.(20) contains an 'ambiguous' term of the type eq.(13), but in the sum they cancel leaving

$$W_1^b = -g^2 C_R (2\pi)^{1-n} \Gamma(-\epsilon) L^\epsilon e^{\frac{i\pi\epsilon}{2}} \frac{2\pi^{1-\frac{\epsilon}{2}}}{\Gamma(1-\frac{\epsilon}{2})} \{ -\frac{2}{\epsilon} 2^{\frac{\epsilon}{2}} + \frac{\epsilon}{2} 2^{\frac{\epsilon}{2}} [\frac{1}{2} \ln^2 2 - \frac{\pi^2}{12}] + \frac{5}{24} \pi^2 \epsilon + \ln 2 \}. \tag{21}$$

W_1^c in eq.(18) is evaluated in the same way as eq.(5), and it gives

$$W_1^c = g^2 C_R (2\pi)^{1-n} \Gamma(-\epsilon) \cos \frac{\epsilon\pi}{2} (2L)^\epsilon \frac{2}{\epsilon} \frac{\Gamma(\frac{1+\epsilon}{2})}{\Gamma(\frac{1}{2})} 2\pi^{1-\frac{\epsilon}{2}}. \tag{22}$$

4. The Complete Contribution from the Triangle

The complete contribution for the triangle Wilson loop to order g^2 is the sum of the self energy graph and the vertex graph. It is

$$W = W_1 + W_2 \tag{23}$$

$$W = -g^2 C_R (2\pi)^{1-n} L^\epsilon \{ \frac{4}{\epsilon^2} + \frac{2i\pi}{\epsilon} - \frac{1}{\epsilon(1-\epsilon)} + \frac{2C}{\epsilon} + i\pi C - \frac{1}{2} C + \frac{1}{2} C^2 - \frac{7}{12} \pi^2 \} 2\pi^{1-\frac{\epsilon}{2}}. \tag{24}$$

The Euler's constant C is the relic of the expansion of Gamma functions in powers of ϵ. Eq.(24) agrees with the result in the Feynman gauge.

A.A. wishes to thank Prof.J.C.Taylor for the invaluable advise.

References

[1] S. Mandelstam, Nucl. Phys. **B213** (1983) 149
[2] G. Leibbrandt, Phys. Rev. **D29** (1984) 1699
[3] A. Andraši and J. C. Taylor, Nucl. Phys. **B310** (1988) 222
[4] O. Piguet and K. Sibold, Nucl. Phys. **B253** (1985) 517
[5] A. Andraši and J. C. Taylor, Nucl. Phys. **B302** (1988) 123
[6] A. Bassetto, I. A. Korchemskaya, G. P. Korchemsky and G. Nardelli, Nucl. Phys. **B408** (1993) 62

Figure Captions

Fig. 1. The self energy diagram with the gluon exchanged on the spacelike Wilson line.
Fig. 2. The vertex graph W_1. The gluon propagates between the Mandelstam lightlike vector n^* and the spacelike Wilson line.